Article

Educational and Psychological Measurement
1–20
© The Author(s) 2016
Reprints and permissions:
sagepub.com/journalsPermissions.nav
DOI: 10.1177/0013164416661823
epm.sagepub.com
SAGE

An examination of Alternative Multidimensional Scaling Techniques

Sofia Papazoglou[1] and Kostas Mylonas[1]

Abstract

The purpose of this study is to compare alternative multidimensional scaling (MDS) methods for constraining the stimuli on the circumference of a circle and on the surface of a sphere. Specifically, the existing MDS-T method for plotting the stimuli on the circumference of a circle is applied, and its extension is proposed for constraining the stimuli on the surface of a sphere. The data analyzed come from previous research and concerns Maslach and Jackson's burnout syndrome and Holland's vocational personality types. The configurations for the same data on the circle and the sphere shared similarities but also had differences, that is, the general item-groupings were the same but most of the differences across the two methods resulted in more meaningful interpretations for the three-dimensional configuration. Furthermore, in most cases, items and/or scales could be better discriminated from each other on the sphere.

Keywords

constrained multidimensional scaling, circular and spherical constraints on MDS solutions, extension of MDS-T on the sphere, quadrant-specific arctangent transformations, burnout syndrome, Self-Directed Search

Introduction

Multidimensional scaling (MDS) aims to uncover the underlying structure in a proximity matrix by producing a simple geometrical model resembling a map, such that

[1]National and Kapodistrian University of Athens, Greece

Corresponding Author:
Sofia Papazoglou, Department of Psychology, National and Kapodistrian University of Athens, Panepistimiopolis, Ilisia, Athens 15784, Greece.
Email: sofiap@ppp.uoa.gr

the distances in the resulting configuration fit the raw data as well as possible (Dillon & Goldstein, 1984). The overall purpose of this article is to apply alternative MDS methods in order to depict the same data on a circular continuum and on a spherical surface and describe them in comparison to each other.

Plotting the stimuli on the circumference of a circle and on the surface of a sphere through MDS are types of constrained MDS analysis. The terms ''constrained MDS'' and ''confirmatory MDS'' (Borg & Groenen, 2005; Cox & Cox, 2001; Heiser & Meulman, 1983) refer to situations when additional constraints are imposed on the configuration, except for general constraints like the dimensionality and the type of analysis (metric or nonmetric; Borg & Groenen, 2005; Borg & Lingoes, 1980). These constraints refer to structural hypotheses or conditions that the derived configuration should satisfy so that the researcher may be able to test for these hypotheses. These can be tested by incorporating them as additional constraints in the MDS analysis. For example, in a study by Sidiropoulou-Dimakakou, Mylonas, and Argyropoulou (2008), Holland's (1985) RIASEC types were constrained to circular arrangement. If the configuration with additional structural constraints is almost as good or better in terms of fit in comparison to a configuration without such constraints, then the hypotheses can be considered compatible with the data (Bentler & Weeks, 1978; Borg & Groenen, 2005; Borg & Lingoes, 1980). External constraints on the MDS configuration can be also useful in a more exploratory context. The derived configuration from an unrestricted MDS analysis may have some rather unattractive properties (Borg & Lingoes, 1980). For example, when the stimuli subjected to MDS analysis are designed to differ in terms of certain attributes and the axes of the unconstrained configuration do not correspond exactly to these attributes, the researcher cannot be certain whether these discrepancies are due to random error or due to some nonrandom effect (Bloxom, 1978). Before interpreting the solution, one may prefer to rescale the data with a constrained method (Borg & Lingoes, 1980). The types of constraints addressed in different MDS methods include equality restrictions on coordinates or distances, fixing parameters to a priori values, or estimating the parameters of an MDS solution so as to yield a specific geometrical structure (Bentler & Weeks, 1978; Bloxom, 1978; Borg & Groenen, 2005; Borg & Lingoes, 1980; de Leeuw & Mair, 2009; Lee, 1984; Lee & Bentler, 1980).

Constraining the MDS solution on a circle or on a sphere can be theoretically meaningful and aid the researcher in further stages of his analyses. In some domains the theoretical structure is supposed to be circular or spherical and discrepancies from it may be considered as ''error.'' Bimler and Kirkland (2005) refer to the different distances of items from the origin in an MDS solution as ''specificity,'' which they claim is approximately constant in a well-designed set of items. The theoretically expected structure is a circular or spherical arrangement, for example, when analyzing color-similarity data (the color circle describing perception of colors with differing wavelength) or similarities between nations (the globe is spherical; Borg & Lingoes, 1980; Cox & Cox, 1991, 2001; de Leeuw & Mair, 2009; Lee & Bentler, 1980). Further examples include models related to Prediger's (1982) hypothesis about the structure of Holland's (1985) vocational personality types (e.g., Rounds & Tracey, 1993), where the theoretical presupposition is that the points lie on the circumference of a circle, the spherical model of vocational interests (Tracey & Rounds, 1996) and circular models of the interpersonal domain (Gurtman & Balakrishnan, 1998). Analysis of circumplex models (e.g., the *circulant model*, the *geometric circulant model*, and the *quasi-circumplex model* as described by Tracey, 2000) is an issue related to circular and spherical MDS structures. Such models can be studied through different methods including structural equation modeling and constrained MDS (e.g., Darcy & Tracey, 2007; Rounds & Tracey, 1993). The gain of constraining the MDS configuration on a circle or on a sphere in cases like the ones just mentioned is that the final solution is closer to the original theory, as the hypothesis of circular or spherical arrangement of points is met, and the similarities and differences of the final solution to the theoretically expected can be meaningfully described. In their paradigm of this, Sidiropoulou-Dimakakou et al. (2008) analyzed data with respect to Holland's hexagonal vocational personality model through MDS. In the unconstrained two-dimensional solution, the arrangement of the RIASEC types was approximately circular, but the different radial distances of the points did not aid interpretation, as the theory presupposes that the six types lie on a circular continuum. Consequently, the axes of this configuration could not be easily interpreted as dimensions. The constrained circular solution was closer to the original theory (the circular property was met), and as a result its similarities and differences to Holland's equilateral hexagon could be described for the specific sample (e.g., the main finding was that the Realistic and Investigative types had much smaller distance than expected).

The circular or spherical constraints can also be used as a way to obtain homogeneous groups of items, stimuli, individuals, and so on, for purposes of better interpreting the solution. Kruskal and Wish (1978) refer to the process of interpreting the derived MDS configuration by grouping stimuli that are close to each other, as neighborhood interpretation (they use cluster analysis applied to the proximity matrix in order to find the item-groups). Another example of a technique for obtaining groups of similar stimuli is Latent Class MDS (Vera, Macías, & Heiser, 2009), where the stimuli are partitioned into classes and the cluster centers are represented in a low-dimensional space. Such grouping of stimuli can also be achieved by locating the nearby points on the circumference of the circle (or on the surface of a sphere) in a constrained circular (or spherical) configuration, and describing their common characteristics that differentiate them from other such groups.

According to Guilford's (1954) homogeneity hypothesis, analyzing homogeneous groups of individuals can lead to bringing out the structure of the data more clearly than when these groups are heterogeneous. This kind of homogeneous groups are useful for bias reduction purposes in comparisons between groups. Thus, when homogeneous groups are compared to each other instead of single units, any similarity or difference that exists between these groups can become apparent because of the reduction of error within the homogeneous groups. For example, in Mylonas et al.

(2011), constrained MDS on a circle is used in order to form homogeneous groups of countries, where the countries comprising each group on the circle had similar factor structures with respect to some construct (work values). These country groups were then tested for factor structure equivalence with respect to another (correlate) construct (Person-Job Fit; Brkich, Jeffs, & Carless, 2002), and this country-group comparison resulted in reduction of ‘‘bias in terms of culture,’’ as opposed to comparison of separate countries where bias was present.

The points of an MDS configuration can be constrained to lie on a circular continuum or on a spherical surface as a special case of imposing constraints (Borg & Lingoes, 1980; Cox & Cox, 1991; de Leeuw & Mair, 2009; Lee, 1984; Lee & Bentler, 1980). The alternative MDS methods for imposing circular and spherical constraints, which are employed in this article, are described in the following sections.

Constraining the MDS Configuration to a Circular Structure and a Spherical Structure

The MDS-T method for constraining a two-dimensional configuration on the circumference of a circle has been proposed by the second author (Mylonas, 2009; Mylonas et al., 2011). Information with respect to the distance of each stimulus from the origin is not used, in order to simplify the patterns present on the configuration and facilitate interpretation. Discrete homogeneous groups of stimuli can become apparent through the grouping of points located on the same or neighboring positions on the circumference. In the MDS-T method, an unconstrained MDS solution in two dimensions is first calculated for a given data set (e.g., with the ALSCAL algorithm; MacCallum, 1981; Takane, Young, & de Leeuw, 1977). The symbols x and y are used to denote the first and second coordinates of a stimulus, respectively. A trigonometric transformation is applied to these coordinates in order to determine the position of the stimuli on the circumference of the circle. This position is determined by computing for each stimulus the angle (clockwise or counterclockwise) defined by the line connecting the respective stimulus with the axes-origin and the positive x-axis. The angles are calculated by employing the quadrant-specific arctangent function for every set of x- and y-coordinates. This trigonometric transformation yields angles expressed in radians with a range of $-\pi$ to π, which are subsequently converted to degrees with a range of -180 to 180 and plotted on the circumference of a circle. The computational procedures as presented in Mylonas (2009) are as follows:

Given that

$$\text{degrees} = \text{radians}\frac{180}{\pi}, \tag{1}$$

$$\text{sgn}(a) = \frac{a}{|a|}, \alpha \in \Re \tag{2}$$

and the formula for computing the tangent of an angle φ

$$\tan(\varphi) = \frac{y}{x}, \tag{3}$$

where x and y the coordinates of a point in two dimensions, the quadrant-specific arctangent value with range $(-\pi, \pi)$ for a stimulus is calculated using the following equations:

$$\text{for } y \neq 0 \text{ and } x > 0, \quad \text{radians}(x, y) = \tan^{-1}\left(\left|\frac{y}{x}\right|\right)[\text{sgn}(y)], \tag{4}$$[1]

$$\text{for } y \neq 0 \text{ and } x = 0, \quad \text{radians}(x, y) = \left(\frac{\pi}{2}\right)[\text{sgn}(y)], \tag{5}$$

$$\text{for } y \neq 0 \text{ and } x < 0, \quad \text{radians}(x, y) = \left\{\pi - \left[\tan^{-1}\left(\left|\frac{y}{x}\right|\right)\right]\right\}[\text{sgn}(y)] \tag{6}$$

$$\text{and for } y = 0 \text{ and } x \neq 0, \text{if } x > 0, \quad \text{radians}(x, y) = 0,$$
$$\text{or if } x < 0, \quad \text{radians}(x, y) = \pi. \tag{7}$$

The derived angles in radians are transformed to degrees and are then plotted on the circumference of a circle. A simple example of the computations involved in the two-dimensional MDS-T method is presented in the appendix.

Extension of the MDS-T Method on a Sphere

In this article, based on the Mylonas' MDS-T method, the first author proposes its extension to three dimensions. In order to plot a specific data set on the surface of a sphere, an unconstrained MDS solution in three dimensions is first computed (e.g., through ALSCAL). If x, y, and z are used to denote the resulting coordinates of a stimulus on the first, the second, and the third dimensions, respectively, these coordinates can be transformed into two angles that determine the location of each stimulus on the spherical surface. The first one of these angles expresses the distance of the stimulus (clockwise or counterclockwise) from the positive x-axis as measured on the circumference of the ''equator'' (similar to longitude). This angle is computed from the x and y coordinates with the MDS-T method for a circle. The second angle is the latitude of the stimulus and expresses the height in which the point is located above or below the ''equator,'' that is, the parallel circle on the circumference of which the stimulus will be plotted. It is the angle defined by the line connecting the stimulus with the center of the sphere and the xy-plane. In order to find the latitude of a point given the Cartesian coordinates x, y, and z, the hemisphere-specific arctangent in radians is computed for the coordinates $(x^2 + y^2)^{1/2}$ and z (Hapgood, 1992). The quantity $(x^2 + y^2)^{1/2}$ is the distance of the projection of a stimulus on the xy-plane from the origin, that is, the distance of the stimulus from the origin if there were only two dimensions. The angle that expresses the latitude of the point ranges from $-1/2\pi$ to $1/2\pi$ radians and can be converted to degrees (with range -90 to 90) when

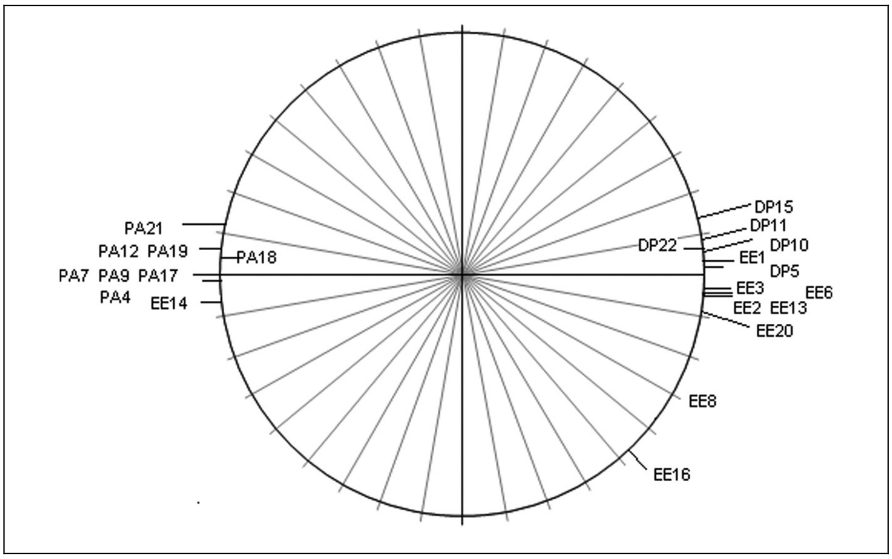

Figure 1. Configuration of the Burnout items on the circumference of a circle.
Note. EE = Emotional Exhaustion; DP = Depersonalization; PA = Personal Accomplishment.

are somewhat higher (.04166 and .05377, respectively) and the R^2 is a little lower (.99275), but they still indicate a good fit, so the circular configuration is acceptable.

The unconstrained three-dimensional MDS solution had also good fit with the Burnout data (Young's *S-stress* = .01257, Kruskal's *stress* = .03344, R^2 =.99653). The *S-stress* and *stress* indices (.06308 and .06318, respectively) are higher for the constrained MDS-T (sphere) solution and the R^2 = .98786 has a somewhat lower value, but they still indicate a good fit, so the sphere is an acceptable configuration. The relative positions of the MBI items are shown in Figure 1 (circle) and Figure 2 (sphere).

Two groups of items were apparent on both configurations, located opposite to each other. The first group is formed by the Personal Accomplishment items and one Emotional Exhaustion item (EE14). The second group consists of the Depersonalization items and most Emotional Exhaustion items. There are two items (EE8 and EE16) that are somewhat separated from the two groups. However, on the circle they seem to be a part of the second group, but on the sphere they are not included in the two homogeneous item groups due to their placement in terms of height.

For the SDS data, the two-dimensional unconstrained MDS solution had a marginal fit with the dissimilarities (Young's *S-stress* = .14716, Kruskal's *stress* = .14588 and R^2 = .91711). The constrained circular MDS-T solution for these items had higher *S-stress* and *stress* values (.20352 and .1722, respectively) and a lower R^2

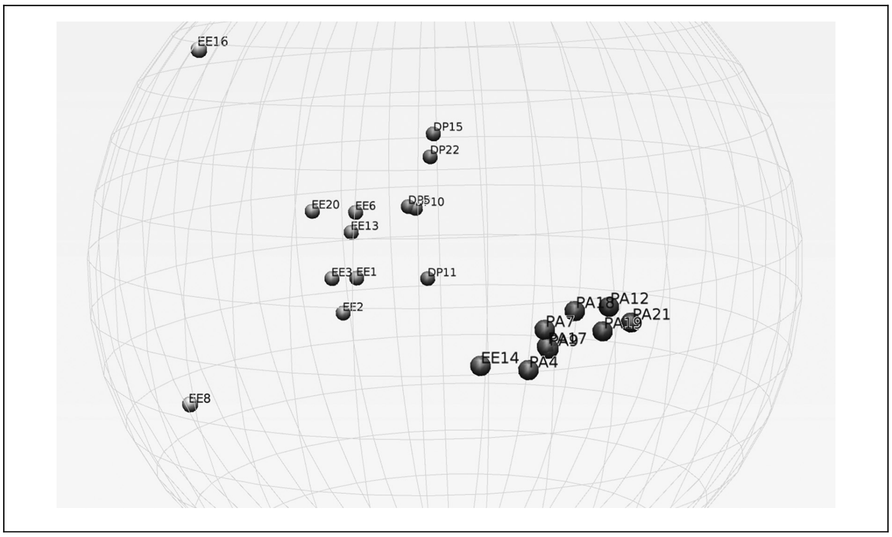

Figure 2. Configuration of the Burnout items on the sphere.
Note. EE = Emotional Exhaustion; DP = Depersonalization; PA = Personal Accomplishment.

value of .88442, so the fit is not very good. The unconstrained three-dimensional MDS solution had adequate fit (Young's *S-stress* = .11843, Kruskal's *Stress* = .10652, R^2 = .94602) and the constrained MDS-T solution on the sphere had somewhat higher *S-stress* and *stress* values (.14963 and .11577) and a somewhat lower R^2 value (.93363), but the deterioration in fit was not very large, so the spherical configuration can be accepted. The angles for the 66 items on the circle and the sphere are presented in Table 2, and the respective plots are shown in Figure 3 (circle) and Figure 4 (sphere).

Examining the placement of the majority of items of each subscale on the circle, it is observed that the configuration resembles a contradistinction between two groups of personality types with the Realistic (R), Conventional (C), and Investigative (I) types forming the first group and the Enterprising (E), Social (S), and Artistic (A) types forming the second group with the R and S types at the opposite ends of a continuum. The types of the first group were very close to each other, and for the second group the S type is in the middle with the A and E types located to its right and the left side respectively.

For the sphere, there seems to be a contradistinction between the Social and Realistic types with most items of these scales located around opposite points. Most items of the C and I subscales lie close to the R type and most items of the A and E subscales are closer to the S type, so there are two groups of types (R-I-C and S-A-E). The placement of items in terms of height gives the impression that the R type is

Finally, Item ‘‘e’’ has a positive x-coordinate and a y-coordinate equal to zero, so according to Equation (7) it corresponds to zero radians (or zero degrees).

Table A. Hypothetical Example for MDS-T in Two Dimensions.

Item	Equation applied	x	y	Radians	Degrees
a	Equation (4)	1.1	2.4	1.14	65
b	Equation (4)	1.4	−2.0	−0.96	−55
c	Equation (5)	0.0	1.2	1.57	90
d	Equation (6)	−1.6	−2.1	−2.22	−127
e	Equation (7)	1.3	0.0	0.0	0

Acknowledgment

We would like to thank the two anonymous reviewers for their perceptive and valuable comments and suggestions.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

Notes

1. For calculation of the trigonometric number $\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}(z)$ rapid convergence is achieved through Euler’s function

$$\tan^{-1}(z)=\sum_{n=0}^{\infty}\frac{2^{2n}(n!)^2}{(2n+1)!}\frac{z^{2n+1}}{(1+z^2)^{n+1}}.$$

2. We would like to thank Giorgos Santipantakis, PhD, postdoctoral researcher at the University of Piraeus, for developing the software program used for plotting the outcomes of our methods. The software for calculating MDS-T 2-dimensional and 3-dimensional solutions and for graphically presenting them on the circle and the sphere can be accessed at psychlabuoa.psych.uoa.gr

References

Bentler, P. M., & Weeks, D. G. (1978). Restricted multidimensional scaling models. *Journal of Mathematical Psychology*, *17*, 138-151.

Bimler, D., & Kirkland, J. (2001). School truants and truancy motivation sorted out with multidimensional scaling. *Journal of Adolescent Research*, *16*(1), 75-102.

Bimler, D., & Kirkland, J. (2003). Smoke and mirrors: Mapping the dimensions of a ‘‘cigarette space’’. *Quality & Quantity*, *37*, 377-391.

Bimler, D., & Kirkland, J. (2005). From circumplex to sphere: Perceptions of vocational activities, explored and applied. *Psihologia Resurselor Umane (Human Resources Psychology)*, *3*(1), 29-40.

Bimler, D., Kirkland, J., Yuhara, N., Kurosaki, M., & Coxhead, E. (2005). ''Trilemmas'': Characterising the Japanese concept of ''amae'' with a three-way forced-ranking technique. *Quality & Quantity*, *39*, 779-800.

Bloxom, B. (1978). Constrained multidimensional scaling in N spaces. *Psychometrika*, *43*, 397-408.

Borg, I., & Groenen, P. J. F. (2005). *Modern multidimensional scaling: Theory and applications* (2nd ed.). New York, NY: Springer.

Borg, I., & Lingoes, J. C. (1980).A model and algorithm for multidimensional scaling with external constraints on the distances. *Psychometrika*, *45*, 25-38.

Brkich, M., Jeffs, D., & Carless, S. A. (2002). A global self-report measure of person-job fit. *European Journal of Psychological Assessment*, *18*, 43-51.

Cox, T. F., & Cox, M. A. A. (1991). Multidimensional scaling on a sphere. *Communications in Statistics: Theory and Methods*, *20*, 2943-2953.

Cox, T. F., & Cox, M. A. A. (2001). *Multidimensional scaling* (2nd ed.). Boca Raton, FL: Chapman & Hall/CRC Press.

Darcy, M. U. A., & Tracey, T. J. G. (2007). Circumplex structure of Holland's RIASEC interests across gender and time. *Journal of Counseling Psychology*, *54*(1), 17-31.

de Leeuw, J., & Mair, P. (2009). Multidimensional scaling using majorization: SMACOF in R. *Journal of Statistical Software*, *31*(3), 1-30.

Dillon, W. R., & Goldstein, M. (1984). *Multivariate analysis. Methods and applications*. New York, NY: John Wiley.

Everitt, B. S. (1996). *Making sense of statistics in psychology. A second-level course*. New York, NY: Oxford University Press.

Gari, A., Mylonas, K., & Panagiotopoulou, P. (2009). Dimensions of social axioms and alternative country-clustering methods. In A. Gari & K. Mylonas (Eds.), *Quod Erat Demonstrandum: From Herodotus' ethnographic journeys to cross-cultural research* (pp. 231-243). Athens, Greece: Pedio Books.

Gari, A., Mylonas, K., & Portešová, S. (2015). An analysis of attitudes towards the gifted students with learning difficulties using two samples of Greek and Czech primary school teachers. *Gifted Education International*, *31*, 271-286.

Guilford, J. P. (1954). *Psychometric methods* (2nd ed.). New York, NY: McGraw-Hill.

Gurtman, M. B., & Balakrishnan, J. D. (1998). Circular measurement redux: The analysis and interpretation of interpersonal circle profiles. *Clinical Psychology: Science and Practice*, *5*, 344-360.

Hapgood, M. A. (1992). Space physics coordinate transformations: A user guide. *Planetary and Space Science*, *40*, 711-717.

Harvey, S. T., Bimler, D., Evans, I. M., Kirkland, J., & Pechtel, P. (2012). Mapping the classroom emotional environment. *Teaching and Teacher Education*, *28*, 628-640.

Heiser, W. J., & Meulman, J. (1983). Constrained multidimensional scaling, including confirmation. *Applied Psychological Measurement*, *7*, 381-404.

Holland, J. L. (1985). *Making vocational choices: A theory of vocational personalities & work environments* (2nd ed.). Englewood Cliffs, NJ: Prentice- Hall.

Kirkland, J., Bimler, D., Drawneek, A., McKim, M., & Schölmerich, A. (2004). An alternative approach for the analyses and interpretation of attachment sort items. *Early Child Development and Care*, *174*, 701-719.

Koliadis, E., Mylonas, K., Koubias, E., Varfi, V., & Valsami, N. (2000). Epagelmatike eksouthenose ekpedeftikon proscholikes kai protovathmias ekpedefsis. [Occupational burnout in pre-school and primary school teachers]. In P. Papoulia-Tzelepi (Ed.), *Literacy in the Balkans: Current situation and future perspective* (pp. 285-293). Athens: Hellenic Association for Language and Literacy.

Kruskal, J. B., & Wish, M. (1978). *Multidimensional Scaling*. Beverly Hills: Sage.

Lee, S.-Y. (1984). Multidimensional scaling models with inequality and equality constraints. *Communications in Statistics: Simulation and Computation*, *13*, 127-140.

Lee, S.-Y., & Bentler, P. M. (1980). Functional relations in multidimensional scaling. *British Journal of Mathematical and Statistical Psychology*, *33*, 142-150.

MacCallum, R. (1981). Evaluating goodness of fit in nonmetric multidimensional scaling by ALSCAL. *Applied Psychological Measurement*, *5*, 377-382.

Maslach, C., & Jackson, S. E. (1986). *Maslach Burnout Inventory: Manual* (2nd ed.). Palo Alto, CA: Consulting Psychologists Press.

Mylonas, K. (2009). Statistical analysis techniques based on cross-cultural research methods: Cross-cultural paradigms and intra- country comparisons. *Psychology, The Journal of the Hellenic Psychological Society*, *16*, 185-204.

Mylonas, K., Gari, A., Panagiotopoulou, P., Georgiadi, E., Valchev, V., Papazoglou, S., & Brkich, M. (2011). Bias in terms of culture: Work values country-clustering for 33 European countries and person-job fit factor equivalence testing for four European countries. In J. Deutch, M. Boehnke, U. Kühnen, & K. Boehnke (Eds.), *Rendering borders obsolete: Cross-cultural and cultural psychology as an interdisciplinary, multi-method endeavour* (pp. 142-157). Bremen, Germany: International Association for Cross-Cultural Psychology.

Prediger, D. J. (1982). Dimensions underlying Holland's hexagon: Missing link between interests and occupations? *Journal of Vocational Behavior*, *21*, 259-287.

Rounds, J., & Tracey, T. J. (1993). Prediger's dimensional representation of Holland's RIASEC circumplex. *Journal of Applied Psychology*, *78*, 875-890.

Sidiropoulou-Dimakakou, D., Mylonas, K., & Argyropoulou, K. (2008). Holland's hexagonal personality model for a sample of Greek university students. *International Journal for Educational and Vocational Guidance*, *8*, 111-125.

Sidiropoulou-Dimakakou, D. (2010). *Epagelmatike aksiologise: test ke erotimatologia epagelmatikou prosanatolismou [Vocational assessment: tests and questionnaires for vocational guidance]*. Athens: Ekdoseis Metaixmio.

Snyder, J. P. (1987). *Map projections—A working manual* (U.S. Geological Survey Professional Paper 1395). Washington, DC: U.S. Government Printing Office.

Stewart, R., Hill, K., Stewart, J., Bimler, D., & Kirkland, J. (2005). Why I am a psychology major: An empirical analysis of student motivations. *Quality & Quantity*, *39*, 687-709.

Takane, Y., Young, F. W., & de Leeuw, J. (1977). Nonmetric individual differences multidimensional scaling: an alternating least squares method with optimal scaling features. *Psychometrika*, *42*, 7-67.

Tracey, T. J. G. (2000). Analysis of circumplex models. In H. E. Tinsley & S. D. Brown (Eds.), *Handbook of applied multivariate statistics and mathematical modeling* (pp. 641-664). San Diego, CA: Academic Press.

Tracey, T. J. G., & Rounds, J. (1996). The spherical representation of vocational interests. *Journal of Vocational Behavior*, *48*, 3-41.

Veligekas, P., Mylonas, K., & Zervas, Y. (2007). Goal orientation and beliefs about the causes of success among Greek track & field athletes. *Perceptual and Motor Skills*, *105*, 927-938.

Vera, J. F., Macías, R., & Heiser, W. J. (2009). A latent class multidimensional scaling model for two-way one-mode continuous rating dissimilarity data. *Psychometrika*, *74*, 297-315.